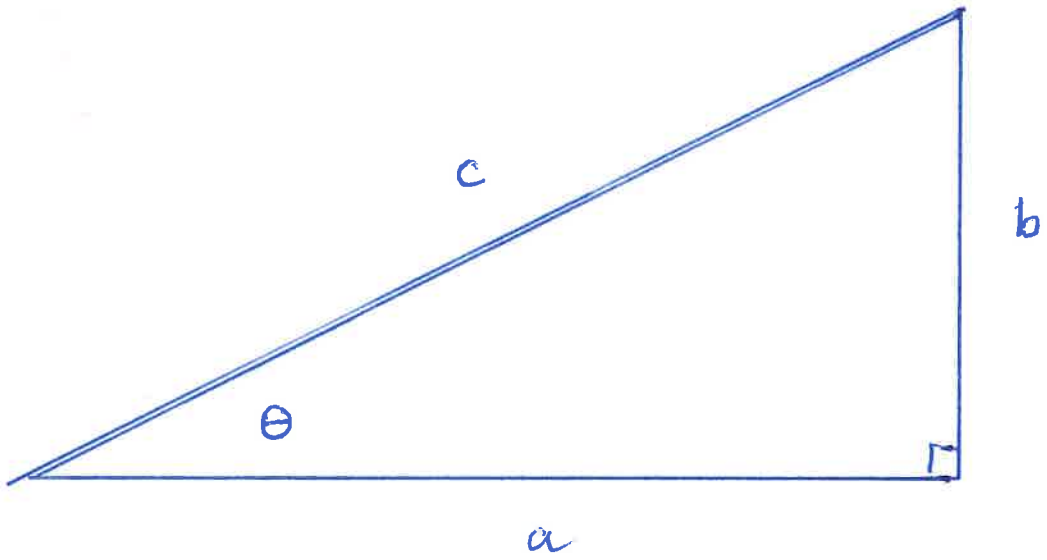
Section 6.2 Trigonometric Functions of Acute Angles

Let θ be an acute angle in a right triangle:

$$\sin\theta = \frac{\text{length of the side opposite } \theta}{\text{length of the hypotenuse}} \qquad \cos\theta = \frac{\text{length of the side adjacent } \theta}{\text{length of the hypotenuse}}$$

$$\tan\theta = \frac{\text{length of the side opposite } \theta}{\text{length of the side adjacent } \theta} \qquad \cot\theta = \frac{\text{length of the side adjacent } \theta}{\text{length of the side opposite } \theta}$$

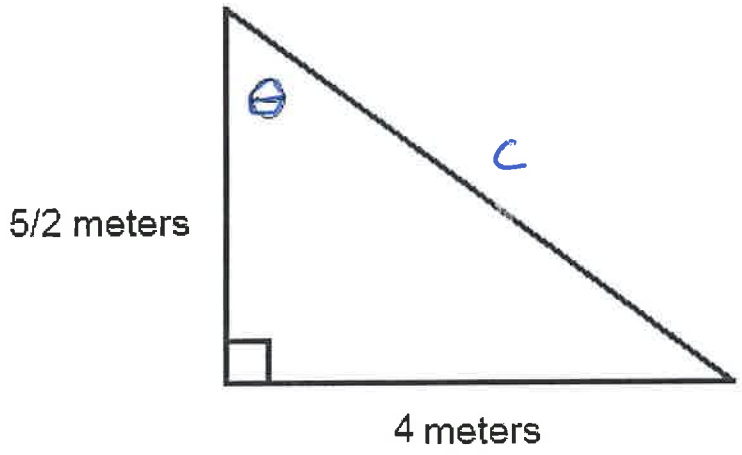
$$\csc\theta = \frac{\text{length of the hypotenuse}}{\text{length of the side opposite } \theta} \qquad \sec\theta = \frac{\text{length of the hypotenuse}}{\text{length of the side adjacent } \theta}$$



$\sin\theta = \frac{b}{c}$, $\tan\theta = \frac{b}{a}$, $\sec\theta = \frac{c}{a}$

$\cos\theta = \frac{a}{c}$, $\cot\theta = \frac{a}{b}$, $\csc\theta = \frac{c}{b}$

Determine the values of the six trigonometric functions of θ



$c^2 = a^2 + b^2$

$c^2 = 16 + \frac{25}{4}$

$c^2 = \frac{89}{4}$

$c = \frac{\sqrt{89}}{2}$

$\sin\theta = \frac{4}{\frac{\sqrt{89}}{2}} = \frac{8}{\sqrt{89}}$

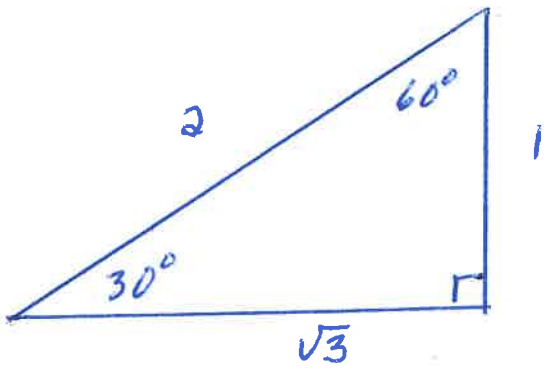
$\cos\theta = \frac{\frac{5}{2}}{\frac{\sqrt{89}}{2}} = \frac{5}{\sqrt{89}}$

$\tan\theta = \frac{4}{5/2} = \frac{8}{5}$

$\csc\theta = \frac{\sqrt{89}}{8}$, $\sec\theta = \frac{\sqrt{89}}{5}$, $\cot\theta = \frac{5}{8}$

Special Triangles:

$30°, 60°, 90°$ $\left(\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right)$ Triangles:



$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

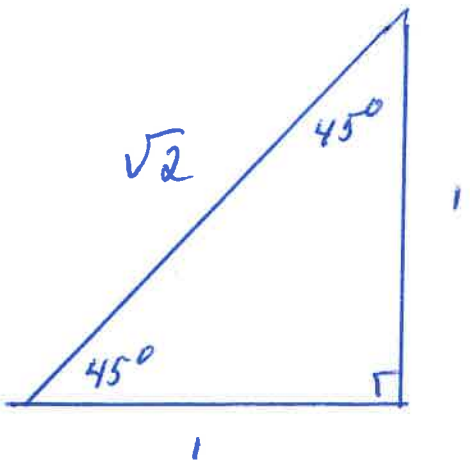
$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc 30° = 2$

$\sec 30° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot 30° = \frac{\sqrt{3}}{1} = \sqrt{3}$

$45°, 45°, 90°$ $\left(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right)$ Triangles:



$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$

$\csc 45° = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\sec 45° = \sqrt{2}$

$\cot 45° = 1$

It is required that you memorize the following trigonometric values:

$$\sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\csc\frac{\pi}{6} = 2 \qquad \sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3} \qquad \cot\frac{\pi}{6} = \sqrt{3}$$

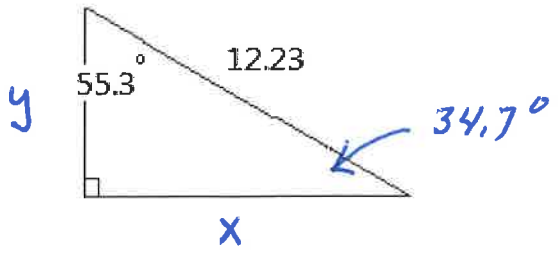
$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2} \qquad \tan\frac{\pi}{3} = \sqrt{3}$$

$$\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3} \qquad \sec\frac{\pi}{3} = 2 \qquad \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \tan\frac{\pi}{4} = 1$$

$$\csc\frac{\pi}{4} = \sqrt{2} \qquad \sec\frac{\pi}{4} = \sqrt{2} \qquad \cot\frac{\pi}{4} = 1$$

Solve the right triangle below:



$\sin(55.3°) = \frac{x}{12.23}$

$x = (12.23) \cdot \sin(55.3°)$

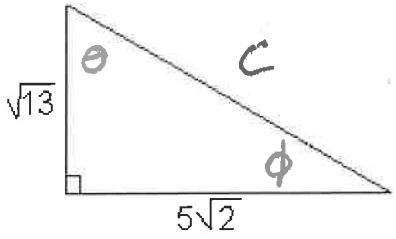
$x = 10.055$

$\sin(34.7°) = \frac{y}{12.23}$

$y = (12.23)\sin(34.7°)$ (3 decimals)

$y = 6.962$

Solve the right triangle below:



$c^2 = 13 + 25 - (2)$

$c^2 = 63$

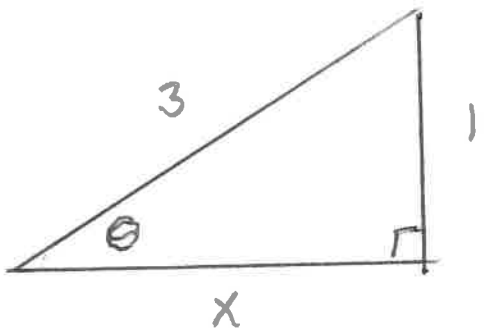
$c = \sqrt{63}$

$\sin\theta = \frac{5\sqrt{2}}{\sqrt{63}} = .89087$

$\theta = 62.98^\circ$

$\phi = 90^\circ - \theta = 90^\circ - 62.98^\circ = 27.02^\circ$

If θ is an acute angle and if $\csc\theta = 3$ then find the other five trigonometric values for the angle θ.



$x^2 + 1 = 3, \quad x^2 = 2$

$x = \sqrt{2}$

$\sin\theta = \frac{1}{3}$

$\tan\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cot\theta = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\sec\theta = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$\cos\theta = \frac{\sqrt{2}}{3}$

Degree, Minute, and Second Notation:

$1' =$ one minute $= \frac{1}{60}^{o}$

$1'' =$ one second $= \frac{1}{60}' = \frac{1}{3600}^{o}$

Write the angle $42^{o}\ 32'\ 14''$ in decimal form rounded to four decimals.

$$42^{o} + \left(\frac{32}{60}\right)^{o} + \left(\frac{14}{3600}\right)^{o}$$

$$= \quad 42.5372^{o}$$